Close Wed: HW_5A,5B,5C (7.1,7.2,7.3)
Office Hours: 1:30-3:00 in Smith 309

Entry Task: Fill in the blanks

Square Identities
$\sin^2(x) =$
$\cos^2(x) =$
$\sec^2(x) =$
$\tan^2(x) =$

Half Angle Identities
$\sin^2(x) =$
$\cos^2(x) =$
$\sin(x)\cos(x) =$

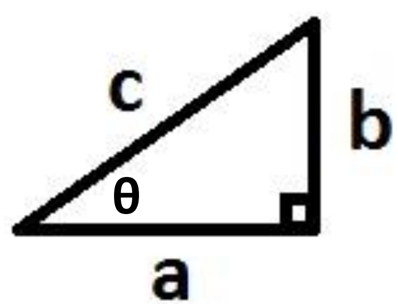
What are these in terms of a, b, and c?

$\sin(\theta) =$

$\cos(\theta) =$

$\tan(\theta) =$

$\sec(\theta) =$



What is the 1st step in each?

(odd sin) $\int \sin^3(x)\cos^4(x)dx$

(odd cos) $\int \sin^5(x)\cos^3(x)dx$

(even sin/cos) $\int \cos^4(x)dx$

Two more cases:

(even sec) $\int \tan^5(\mathrm{x})\sec^4(\mathrm{x})dx$

(odd tan) $\int \tan^5(\mathrm{x})\sec(\mathrm{x})dx$

7.3 Trigonometric Substitution

Goal: Develop a method to evaluate integrals involving expressions of the form $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, or $\sqrt{x^2 - a^2}$

CASE	**SUBSTITUTION**
$a^2 - x^2$	x = a sin(θ), -π/2 ≤ θ ≤ π/2
$a^2 + x^2$	x = a tan(θ), -π/2 < θ < π/2
$x^2 - a^2$	x = a sec(θ), 0 ≤ θ < π/2, (pos. x) π ≤ θ < 3π/2 (neg. x)

Trigonometric Substitution Method:

A) Substitute, don't forget dx = ??dθ. Simplify (eliminate root)
B) Use 7.2 methods for trig integrals.
C) Draw a triangle and return to x.

Quick Examples

$$\int \frac{x^3}{\sqrt{4 - x^2}} dx$$

$$\int x^2 \sqrt{9 + x^2} dx$$

$$\int \frac{\sqrt{x^2 - 16}}{x}\,dx$$

Completing the Square:

$$\sqrt{ax^2 + \boldsymbol{bx} + c}$$

If you ever encounter a "**middle term**" (like **bx** above), then you need to complete the square.

Example: $\sqrt{64 - 24x - 4x^2}$

i) **Factor out the "a".**

$$\sqrt{4(16 - 6x - x^2)} = 2\sqrt{16 - 6x - x^2}$$

ii) **Add and subtract "half the middle squared"**

Half of middle = (-6)/2 = -3

Squared = $(-3)^2$ = 9

$$2\sqrt{16 + 9 - 9 - 6x - x^2}$$

iii) **Factor the perfect square**

$$2\sqrt{25 - (x + 3)^2}$$

iv) **Check your work!**